

Core loss and excitation current model for wound core distribution transformers

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SUMMARY

This paper proposes a model for the computation of core losses and excitation current, in a lamination by lamination method, for wound core distribution transformers. The model was developed based on the finite-element method (FEM). The results obtained by applying the proposed model were compared with the FEM results and with the measurements of the no-load test. The no-load losses obtained by the proposed model present a difference of 4% with respect to measured values, while they are almost the same with respect to FEM. The proposed model contributes in the research of new techniques that improve transformer design. Copyright © 2012 John Wiley & Sons, Ltd.

KEY WORDS: core losses; eddy current losses; finite-element method; magnetic flux density; magnetic materials; no-load test; wound core; transformer design

1. INTRODUCTION

Increasing competition in the global transformer market has put tremendous responsibilities on the industry to improve transformer design while reducing cost [1–4]. Improving the estimation of excitation current and component losses in the transformer core has attracted the interest of many researchers and manufacturers; diverse analytical equations have been found and applied to compute core losses by using a systematic procedure for the determination of the incremental self- and mutual inductances of the windings [5], or the magnetic energy by applying Poynting's theorem [6]–[7]. In the last decades with advancements in computing capabilities, techniques based on numerical analysis of electromagnetic effects were developed to determine winding and core losses [8–11]. Another important contribution in the computation of core loss is based on the equivalent conductivity [12]–[13]. Core loss computation is a nonlinear problem, and it requires a numerical iterative procedure to solve it, either in time or in frequency domain, which consumes great computational memory. This paper contributes to the estimation of core losses on the design stage, modeling this problem in a simplified and efficient manner, thus making easy and fast the calculations during the transformer design process. Besides, the model provides very important information in the core joint zone of the laminations; in particular, it provides the highest value of magnetic flux density B as well as its location in the laminations. The model also considers the number of laminations per step, which is a design parameter that affects the magnetic flux density and the excitation current in the core. To achieve the above-mentioned advantages, it is necessary to apply a Gaussian model (GM) for the magnetic flux distribution obtained by applying the inductive method using simulations with finite-element method (FEM)

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performed in 3D and 2D. The results obtained with the proposed model are compared with FEM as well as the values extracted from the no-load test.

2. WOUND CORE GEOMETRY AND CONSIDERATIONS

This section describes the geometry and the design parameters considered in this work. The simulations implemented consider the conventional wound core with shell-type configuration, as well as the octagonal wound core shape with shell-type configuration [13]; however, the model can be also applied to the core type configuration. In this work, the model is applied for a single-phase transformer with three limbs, but it could be also adapted for the three-phase wound core transformer with five legs. Figure 1a shows the geometric model for the laminations and the number of laminations per step nl . Figure 1b shows the design parameters considered, such as overlap length s , air-gap length g , and lamination thickness d [14]. Other parameters shown in Figure 1b are lamination width w , window height h , and window width u .

In order to obtain the eddy current losses in wound core laminations, the no-load test was simulated by a time-harmonic field simulation using FEM. The 2D FEM simulations were executed for all the laminations of the wound core using the 2D geometric model for each lamination. On the other hand, the 3D FEM simulations were executed for groups of no more than 20 laminations, because the simulation of all the laminations in 3D means run out of our computational memory (see Appendix). The results obtained for the eddy current losses computed with FEM in 2D and 3D were very similar, with less than 2% difference. Thus, we considered that computing the eddy current losses in 2D could be enough, but it could be poor to show the distribution of B in the laminations. In this work, four different grain-oriented silicon steel (GOSS) grades were used to simulate and compute their losses: M4 (0.28 mm), M5 (0.30 mm), M6 (0.35 mm), and M5-H2 (0.30 mm) [15].

3. PROPOSED MODEL

3.1. Magnetic flux density distribution model

The proposed model determines the core losses in each lamination of the core; that is why it was necessary to obtain a model for the distribution of the normal magnetic flux density B . The distribution of B in the wound core is shown in Figure 2a; values of B in the xy plane along one randomly selected lamination were extracted and plotted in Figure 2b, practically the same shape of B along the lamination was found for the other core laminations. From the analysis of the shape of B in Figure 2b, two zones are observed: (i) the zone where B is non-uniform (joint zone, λ); (ii) the zone where B is steady or uniform (region ℓ).

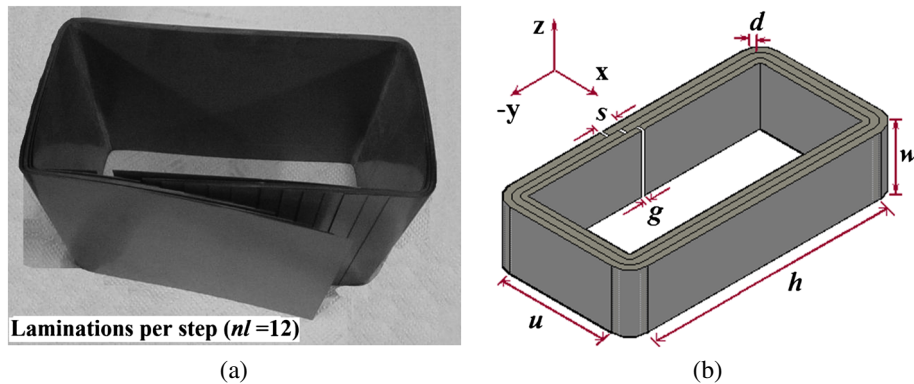


Figure 1. (a) Laminations per step of a wound core distribution transformer. (b) Design parameters considered. More details can be found in [14].

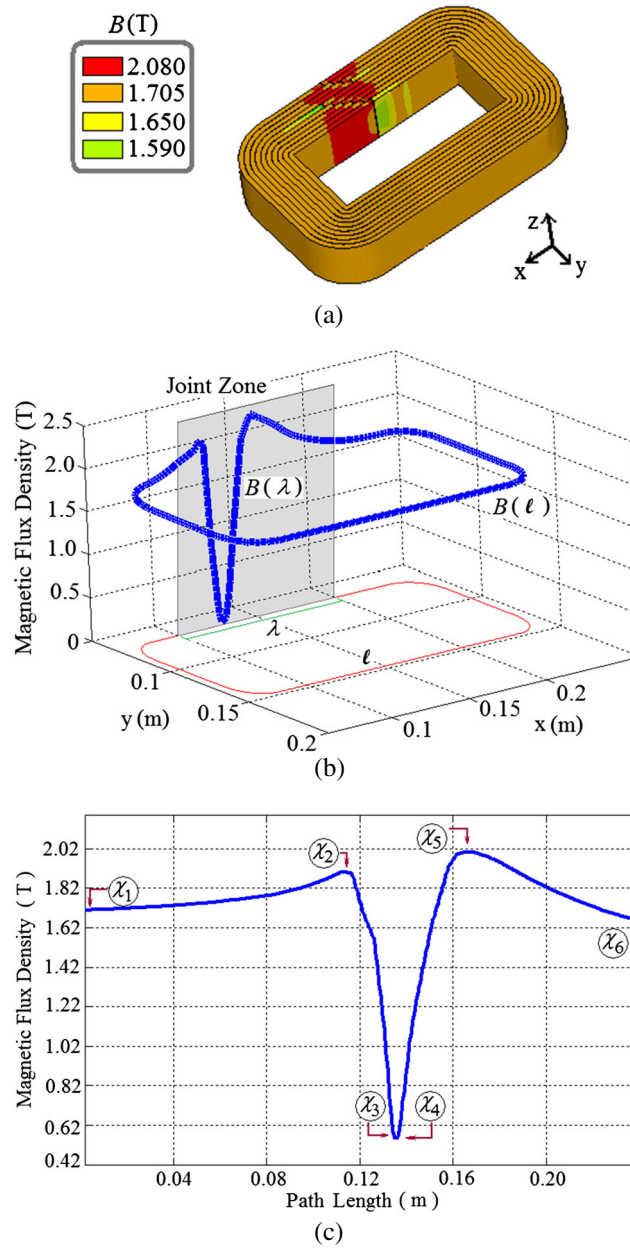
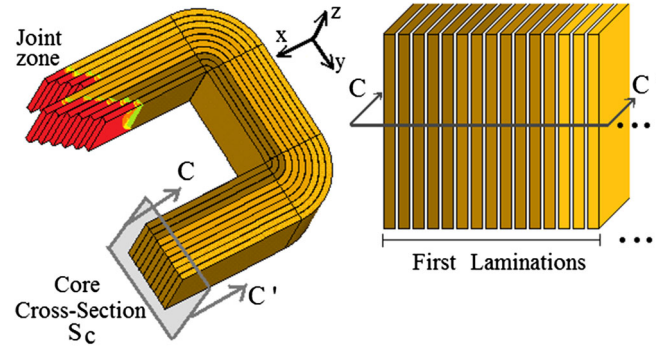


Figure 2. (a) Distribution of magnetic flux density in the wound core. (b) Behavior of magnetic flux density B along one lamination. (c) Behavior of B in the joint zone along one lamination.

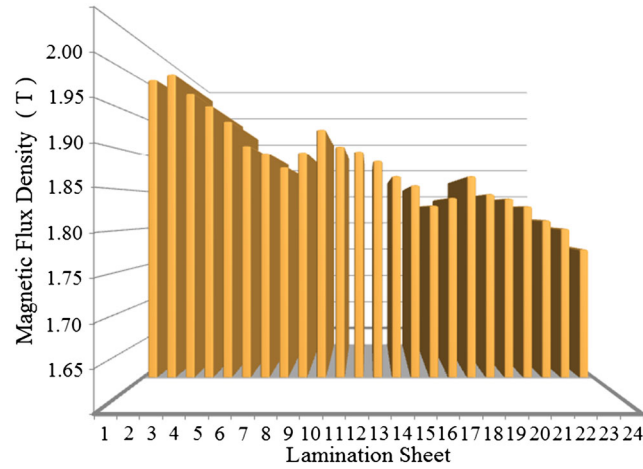
For the region where B is non-uniform, it was necessary to find a model that fits with magnetic flux density path (Figure 2c); it was fitted with a GM [16].

For the uniform region, it is necessary to consider the B values in the yz plane, i.e. the core cross-section plane (Figure 3a). The distribution of B along this plane yz is shown in Figure 3b, where it is possible to notice that the laminations close to the core window have higher B value than external laminations. Thus, in order to compute the average of the magnetic flux density B_{0k} values in each lamination, it is necessary to find a model that fits with Figure 3c; for this, we assumed that B_{0k} in yz plane (the core cross-section plane) varies as an exponential function of the independent variable τ that represents the distance from zero core width until the core width E (Figure 3c). Thus:

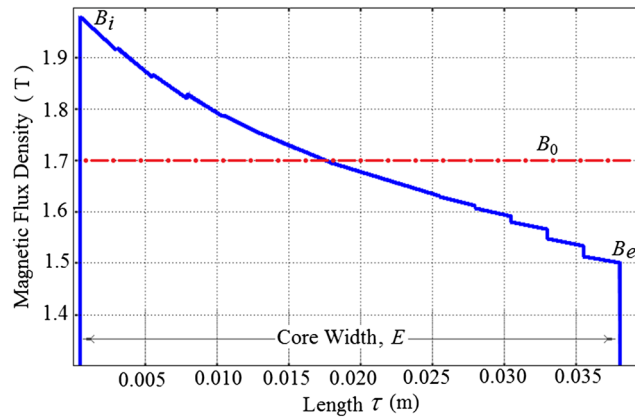
$$B_{0k}(\tau) = K \cdot e^{M \cdot \tau} \quad (1a)$$



(a)



(b)



(c)

Figure 3. (a) Cross-section where the magnetic flux density shape is analyzed. (b) Magnetic flux density values in the first 24 laminations. (c) Magnetic flux density shape in the cross-section plane.

$$B_{0k}(0) = K \cdot e^{M \cdot 0} = K = B_i \quad (1b)$$

$$B_{0k}(E) = K \cdot e^{M \cdot E} = B_e \quad (1c)$$

where B_i is the magnetic flux density in the internal lamination where $\tau=0$ and B_e is the magnetic flux density in the external lamination where $\tau=E$. The constant M can be obtained by:

$$M = \frac{1}{E} \cdot \ln\left(\frac{B_{0k}(E)}{K}\right) = \frac{1}{E} \cdot \ln\left(\frac{B_e}{B_i}\right) = \frac{1}{E} \cdot \ln\left(\frac{\ell_i}{\ell_e}\right) \quad (2)$$

where Ln is the natural logarithm and the last term of Equation (2) was obtained considering that reduction of B is because the internal laminations have less reluctance than the external laminations because they are shorter than external laminations.

The reluctances, \mathfrak{R}_i and \mathfrak{R}_e , in the internal and external lamination, respectively, are given by:

$$\mathfrak{R}_i = \frac{mmf}{\phi_i} = \frac{mmf}{B_i \cdot A_s} = \frac{\ell_i}{\mu_0 \cdot \mu_r \cdot A_s} \quad (3a)$$

$$\mathfrak{R}_e = \frac{mmf}{\phi_e} = \frac{mmf}{B_e \cdot A_s} = \frac{\ell_e}{\mu_0 \cdot \mu_r \cdot A_s} \quad (3b)$$

where mmf is the magnetomotive force; ϕ_i and ϕ_e is the flux in the internal and external lamination, respectively; B_i and B_e are the magnetic flux density in the internal and external laminations, respectively; ℓ_i and ℓ_e are the length of the internal and external laminations, respectively; A_s is the cross-section area of the lamination, and its relative permeability is μ_r .

Considering that the magnetomotive force mmf is the same (since the ampere-turns of the excitation source is the same), using Equations (1a) and (1b), the following relation is obtained:

$$\frac{\mathfrak{R}_i}{\mathfrak{R}_e} = \frac{B_e}{B_i} = \frac{\ell_i}{\ell_e} \quad (4)$$

Thus, the reluctances ratio ($\mathfrak{R}_i/\mathfrak{R}_e$) is equal to the length ratio (ℓ_i/ℓ_e) of the lamination, where ℓ_i and ℓ_e are known from the geometry model.

In the design stage of the wound core transformer, it is common to assume an average value of B in the core cross-section; we refer to this average value as design magnetic flux density and we denote it with B_0 . It is possible to obtain B_0 by applying the average definition to the exponential model established in Equation (1a) as follows:

$$B_0 = \frac{1}{(E-0)} \cdot \int_0^E B_i \cdot e^{M \cdot \tau} d\tau = \frac{B_i}{\text{Ln}(\ell_i) - \text{Ln}(\ell_e)} \cdot \left(\frac{\ell_i}{\ell_e} - 1 \right) \quad (5)$$

Since B_0 is the known (desired) value of the magnetic flux density for the core cross-section, it is possible to determine B_i and B_e as follows:

$$B_i = \frac{B_0 \cdot \ell_e}{(\ell_i - \ell_e)} \cdot \text{Ln} \left(\frac{\ell_i}{\ell_e} \right) \quad (6a)$$

$$B_e = \frac{B_0 \cdot \ell_i}{(\ell_i - \ell_e)} \cdot \text{Ln} \left(\frac{\ell_i}{\ell_e} \right) \quad (6b)$$

Another parameter that was considered is the number of laminations per step nl , which affects the modeling of the distribution of B . The effect of the number of laminations per step (nl) can be incorporated into the proposed exponential model Equation (1a) as:

$$B_{0k}(\tau) = B_i \cdot e^{M \cdot \tau} \cdot \left(1 + \frac{1}{nl} \right) \quad (7)$$

where the term $(1 + 1/nl)$ was obtained after a numerical fit process with all the results obtained from the simulations executed.

3.2. Core loss calculation model

The computation of the core losses in transformer involves the computation of three components: hysteresis losses, P_h (W); classical eddy current losses, P_e (W); and excess losses, P_{exc} (W). In this work, the hysteresis losses were computed in each lamination by applying the manufacturer factor k_h ; for example, for M4, $k_h = 0.3161$ W/kg at 1.50 T and 60 Hz. It should be noted that k_h is not constant, but it depends on the operation frequency (f) and the magnetic flux density (B) at which the core laminations are operating, so $k_h(f, B)$. In this case, the GOSS manufacturer provides k_h -factor only for typical transformer operation frequency (50 and 60 Hz) and B magnitudes (1.5 and 1.7 T) for the different quality steels. The hysteresis loss for the k -th lamination is given by:

$$P_h^{(k)} = k_h \cdot d \cdot w \cdot l_k \cdot \rho_s \quad (8)$$

where d is the lamination thickness (m), w is the lamination width (m), and l_k is the lamination length (m) for the k -th lamination and $\rho_s = 7650 \text{ kg/m}^3$ is the specific weight of the GOSS.

The proposed formula for the computation of the total hysteresis loss is the following:

$$P_h = \sum_{k=1}^{nk} P_h^{(k)} \quad (9)$$

where nk is the total number of laminations in the core.

The classical formulation to determine the eddy current losses is given by [17]:

$$W_e = \frac{1}{6} \cdot \sigma \cdot \pi^2 \cdot f^2 \cdot d^2 \cdot B_0^2 \cdot F(\delta) \quad (10a)$$

The depth factor $F(\delta)$ depends on the penetration depth δ ; $F(\delta)$ is given by:

$$F(\delta) = \frac{3}{\delta} \frac{\sinh(\delta) - \sin(\delta)}{\cosh(\delta) - \cos(\delta)} \quad (10b)$$

where δ is given by:

$$\delta = \frac{d}{\sqrt{\frac{1}{\pi f \sigma \mu_0 \mu_r}}} \quad (10c)$$

For example, the GOSS lamination M5 has the following characteristics: $d = 0.3 \times 10^{-3} \text{ m}$, $\sigma \approx 2.0833 \times 10^6 \text{ S/m}$; $\mu_r \approx 2300$; then, $\delta(50 \text{ Hz}) = 0.92$, $\delta(60 \text{ Hz}) = 1.01$; so, $F(\delta) = 0.9989$ at 50 Hz and $F(\delta) = 0.9983$ at 60 Hz. Consequently, $F(\delta) \approx 1$, for the case of low frequency (e.g. 50 Hz or 60 Hz), in which the transformer operates. That is why the depth factor $F(\delta)$, which appears in Equation (10a), was omitted in this work.

In order to compute the eddy current losses in a lamination-by-lamination arrangement, as we propose in this work, Equation (10a) has to be modified as follows:

$$P_e^{(k)} = \frac{1}{6} \cdot \sigma \cdot \pi^2 \cdot f^2 \cdot d^3 \cdot w \cdot l_k \cdot B_{0k}^2 \quad (11)$$

where $P_e^{(k)}$ is the classical eddy current losses (W) in the k -th lamination in the uniform region l , σ is the conductivity of lamination (S/m), f is the operation frequency (Hz), d is the lamination thickness (m), w is the lamination width (m), l_k is the lamination length (m) in the k -th lamination, B_{0k} is the average value of the magnetic flux density in the k -th lamination (T). The eddy current losses in the joint zone can be estimated as follows:

$$P_{ej}^{(k)} = \frac{1}{6} \sigma \pi^2 f^2 d^3 w \left[\sum_{j=1}^{nj} (B(\lambda))^2 \cdot \Delta \lambda_j \right] \quad (12)$$

where $B(\lambda)$ in the joint zone is given by a GM approximation:

$$B(\lambda) = \alpha_{1,i} \cdot e^{-\left(\frac{\lambda - \beta_{1,i}}{\gamma_{1,i}}\right)^2} + \alpha_{2,i} \cdot e^{-\left(\frac{\lambda - \beta_{2,i}}{\gamma_{2,i}}\right)^2} \quad (12a)$$

where $\alpha_{1,i}$, $\alpha_{2,i}$ are coefficients related to the peak of B ; $\beta_{1,i}$, $\beta_{2,i}$ are coefficients related to the position of the peak of B ; and $\gamma_{1,i}$, $\gamma_{2,i}$ are related to the width of the peak of B . The subscript i represents the section number (from 1 to 5) that we divided the joint zone. The coefficient values in Equation (12) and the process to compute them is given in [16].

The subscript j in Equation (12) represents the number of partitions $\Delta \lambda$ of the joint zone length (λ), nj is the total number of partitions, and k denotes the k -th lamination.

The proposed formula for the computation of the total eddy current losses is the following:

$$P_e = \sum_{k=1}^{nk} \left[P_{ej}^{(k)} + P_e^{(k)} \right] \quad (13)$$

The excess losses in the transformer core can be computed by [18]:

$$W_{exc} = 8.76 \cdot \sqrt{\sigma \cdot G \cdot V_0 \cdot S} \cdot B_0^{1.5} \cdot f^{1.5} \quad (14)$$

where W_{exc} are the excess losses (W/m³), σ is the conductivity of lamination (S/m), G and V_0 are constants related to the material lamination quality ($G=0.1356$ and $V_0=0.0110$ for the M4 magnetic material at 1.50 T and 60 Hz, V_0 is fitting parameter, since it should be fitted for each magnetic induction), and S is the cross-sectional area of the lamination.

The following formula is proposed for computing the excess losses in each lamination:

$$P_{exc}^{(k)} = 8.76 \cdot \sqrt{\sigma \cdot G \cdot V_0 \cdot S} \cdot B_0^{1.5} \cdot f^{1.5} \cdot d \cdot w \cdot l_k \quad (15)$$

where $P_{exc}^{(k)}$ are the excess losses (W) for the k -th lamination with lamination length l_k (m). The proposed formula for the computation of the total excess losses is the following:

$$P_{exc} = \sum_{k=1}^{nk} P_{exc}^{(k)} \quad (16)$$

The total core loss is calculated by:

$$P_c = P_h + P_e + P_{exc} \quad (17)$$

where the component losses P_h , P_e , and P_{exc} are computed by the proposed formulas Equations (9), (13), and (16), respectively.

3.3. Excitation current model

The estimation for the excitation current is obtained after the computation of the lamination reluctance. The magnetic circuit for the wound core is shown in Figure 4: two reluctances are computed in each lamination according with the two zones assumed in this work: (i) Reluctance \mathfrak{R}_λ in the joint zone λ where the magnetic flux density φ_λ is non-uniform; (ii) Reluctance \mathfrak{R}_ℓ in the zone where the magnetic flux φ_ℓ is constant.

For the zone ℓ where the magnetic field is constant, the reluctance can be obtained by:

$$\mathfrak{R}_\ell^{(k)} = \frac{\ell_{(k)}}{\mu_0 \cdot \mu_r \cdot A_s} \quad (18)$$

where $\mathfrak{R}_\ell^{(k)}$ is the reluctance in the k -th lamination with length $\ell_{(k)}$, cross-section A_s , while the relative permeability μ_r is given by the manufacturer in the rolling direction [15]. For the joint zone where the

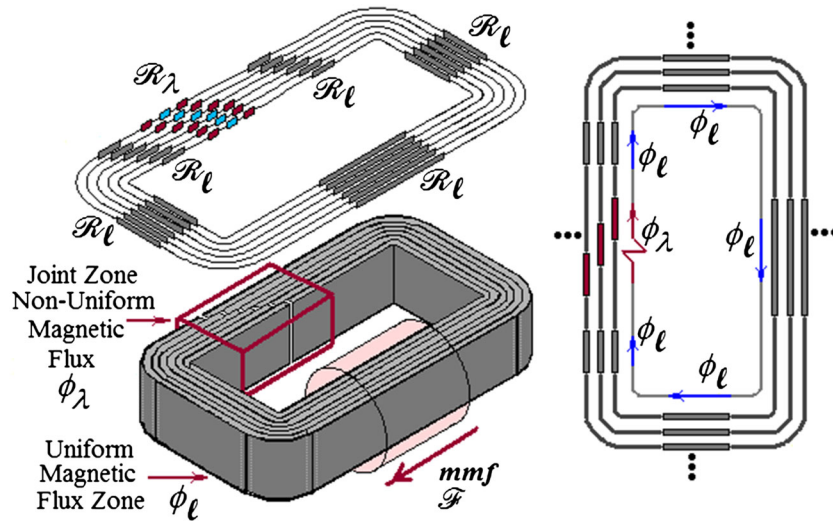


Figure 4. Core reluctance model for the wound core transformer.

magnetic field is non-uniform, the reluctance can be estimated by applying the permeability in the joint zone $\mu(\lambda)$ [16], thus:

$$\mathfrak{R}_{\lambda}^{(k)} = \frac{1}{\mu_0 \cdot A_s} \cdot \left[\frac{\Delta \lambda}{\mu(\lambda)} \right] \quad (19)$$

The total reluctance per lamination is given by:

$$\mathfrak{R}^{(k)} = \mathfrak{R}_{\ell}^{(k)} + \mathfrak{R}_{\lambda}^{(k)} \quad (20)$$

Thus, the following formula is proposed for the computation of the excitation current (ampere-turn):

$$I_{exc} = \left(\sum_{k=1}^{nk} \frac{1}{\mathfrak{R}^{(k)}} \right) \cdot B_0 \cdot S_c \quad (21)$$

where nk is the total number of laminations in the core and S_c is the core cross-sectional area.

4. RESULTS

This section presents the application of the proposed model for the computation of core losses and excitation current, using as examples a single-phase 25 kVA as well as a three-phase 750 kVA distribution transformer. Both transformers have shell-type wound cores. The single-phase transformer, having two identical cores, is composed of three limbs. The three-phase transformer, having two central cores and two lateral cores, is composed of five limbs. The central and lateral cores are identical with the exception that the window width of the central core is twice that of the lateral core [1]. Table I contains the core dimensions and construction parameters for these two transformers.

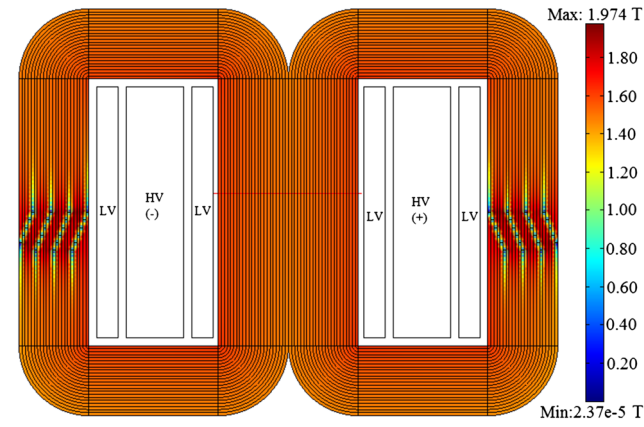
From FEM simulations, Figure 5a shows the distribution of B on the transformer core when the no-load test was simulated; values of B were extracted on two different paths with the goal to evaluate the distribution of B : (i) in the right limb; (ii) in the central limb. Figure 5b shows B on the right limb, where it is possible to notice the decreasing slop of B : the laminations close to the core window present the highest values of B . Figure 5c shows values of B on the central limb: the average value of B in the core cross-section is 1.5 T, since $B_0 = 1.5$ T. Figure 6 shows the distribution of B in the joint zone.

No-load test was performed on these two distribution transformers, and the total core losses as well as the excitation current were extracted from this routine test. Table II contains the summary of core losses and excitation current measured. The designed wound cores have 6 and 12 laminations per step ($nl = 6$ and $nl = 12$); M4 GOSS was used; the design was built to operate at $B_0 = 1.50$ T and 60 Hz.

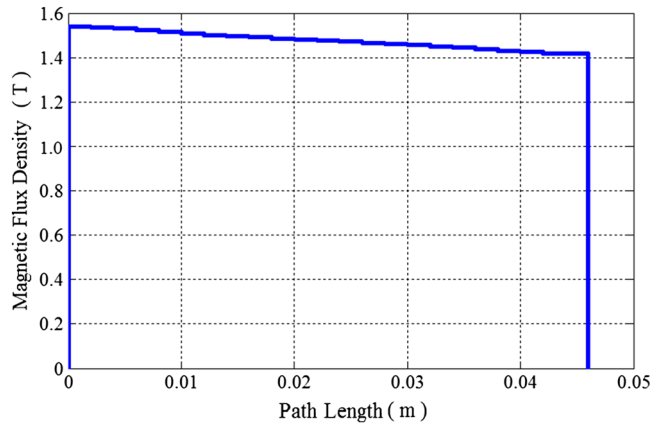
The hysteresis losses P_h were constant for $nl = 6$ and $nl = 12$ (Table II) because we use the only factor given by the manufacturer ($k_h = 0.3161$ at 1.5 T and 60 Hz) in conjunction with Equation (8). The eddy current losses $P_{e,FEM}$ were computed using FEM and the difference with the eddy current losses P_e

Table I. Core construction parameters for the single-phase 25 kVA transformer as well as the three-phase 750 kVA transformer.

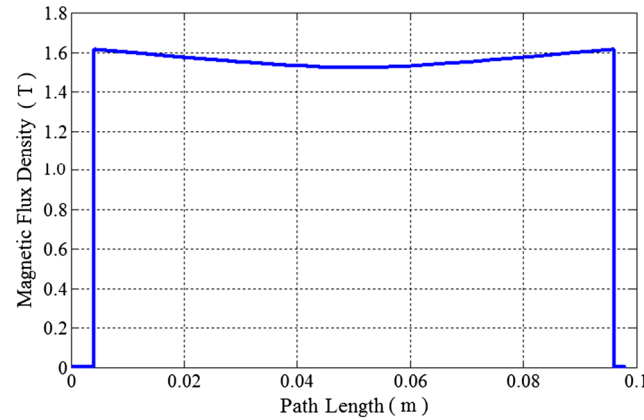
Symbol	Parameter	Single phase	Three phase
B_0	Design magnetic flux density (T)	1.50	1.75
I_p	Primary current (A)	3.28	32.80
E	Core width (mm)	46.00	60.00
h	Window height (mm)	175.00	280.00
u	Window width of lateral core (mm)	85.00	82.80
w	Lamination width (mm)	152.40	304.80
d	Lamination thickness (mm)	0.28	0.28
sil	Space between laminations (mm)	0.02	0.02
s	Overlap length (mm)	10.00	10.00
g	Air gap length (mm)	1.00	1.00
nl	Number of laminations per step	6	8
nk	Total number of laminations	153	200



(a)



(b)



(c)

Figure 5. (a) Distribution of magnetic flux density B on the wound core. (b) Values of B along a path parallel to y axis on the right limb. (c) Values of B along a path parallel to y axis on the central limb.

computed by the proposed formula Equation (13) was around 4% (Table II). The difference between the measurement values and the values of the proposed model is in the order of 4% for the total core losses, but for the excitation current is about 7.5%. A comparison between the calculated values of total core losses obtained by Equation (17), and the measured values for ten different transformers is shown in Figure 7. Consequently, the GM [16] for the magnetic field density improves the accuracy to compute the eddy current losses as well as the total core losses. Furthermore, because of its good

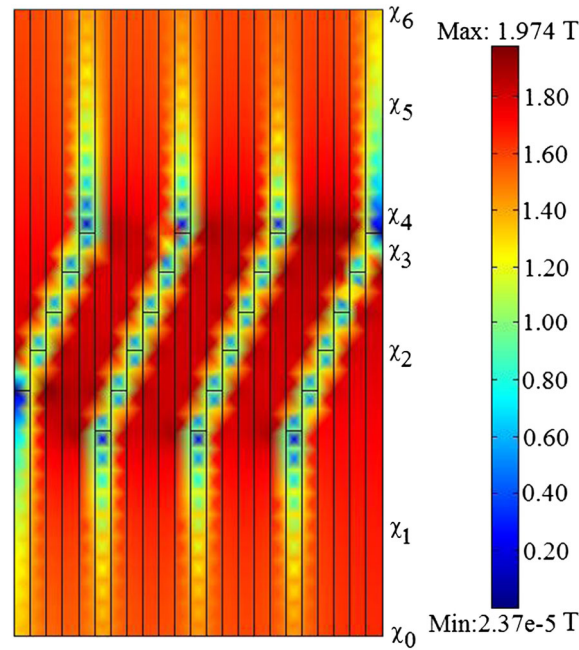


Figure 6. Distribution of magnetic flux density (B) in the joint zone.

Table II. Core losses and excitation current for the single-phase 25 kVA transformer as well as the three-phase 750 kVA transformer.

Symbol	Parameter	Single phase		Three phase
		$nl = 6$	$nl = 12$	
W_{Test}	Core losses (W) from test	81.79	71.62	839.87
P_c	Core losses (W) by proposed model Equation (17)	85.30	73.81	869.08
P_h	Hysteresis losses (W) computed by Equation (9)	21.40	21.40	273.40
P_e	Eddy current losses (W) considering the joint zone by Equation (13)	24.43	19.12	291.70
$P_{e, FEM}$	Eddy current losses (W) considering the joint zone by FEM	25.26	18.60	281.30
P_{exc}	Excess losses (W) computed by Equation (16)	39.47	33.29	303.98
I_p	Primary current (A) from test	3.28	3.27	32.68
$I_{exc, test}$	Excitation current (A) from test	0.0157	0.0144	1.36
I_{exc}	Excitation current (A) computed by Equation (21)	0.0172	0.0156	1.43

accuracy, the GM is able to help in the core design by estimating the eddy current losses as a function of the number of laminations per step nl . The eddy current losses as a function of nl are shown in Figure 8, where the values of the proposed model are compared with FEM solution. In Figure 8, the values are given in per unit (pu), where the base is the result from FEM using a single wound core with ten laminations per step. The results show that as the number of laminations increases, the eddy current losses are decreased. Excitation current as a function of nl is shown in Figure 9, where the same behaviour as the eddy currents losses can be noticed, i.e. as the number of laminations increases, the excitation current is decreased.

5. CONCLUSIONS

This work proposes a model for the computation of core losses and excitation current, which is very useful during the early stage of transformer design. The model, which is computed in a lamination by lamination manner, is based on a GM for the distribution of magnetic flux density. The article also

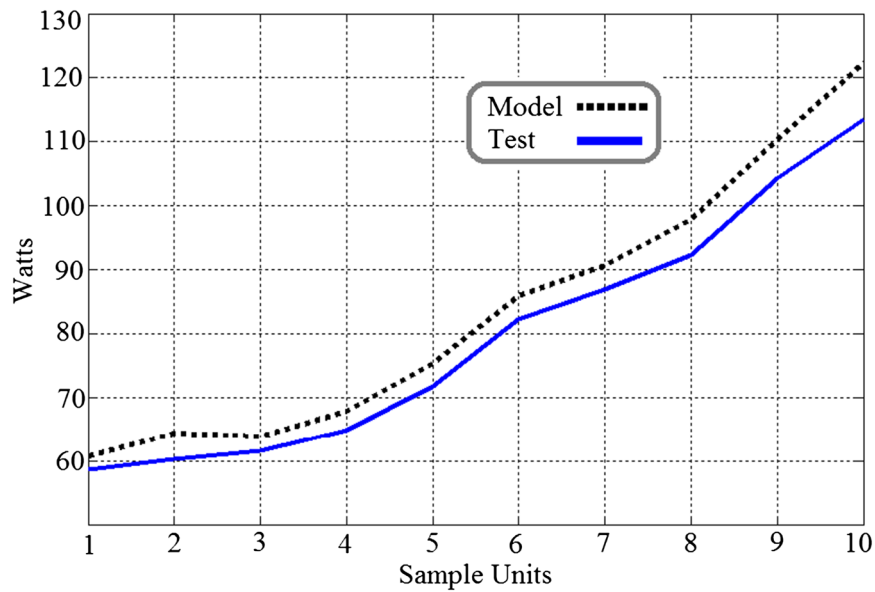


Figure 7. Comparison of calculated and measured values of total core losses for ten different transformers.

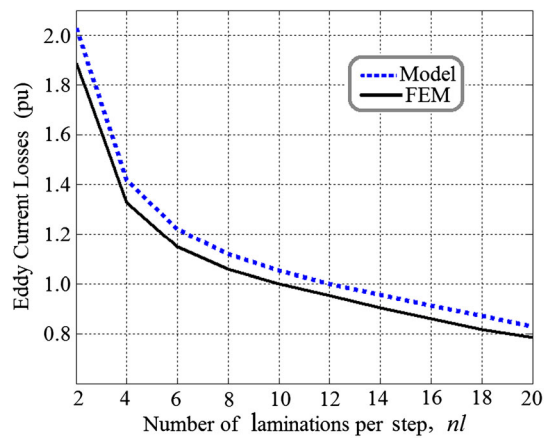


Figure 8. Eddy current loss results by applying FEM as well as the proposed model Equation (13) when the number of laminations per step (nl) is changing.

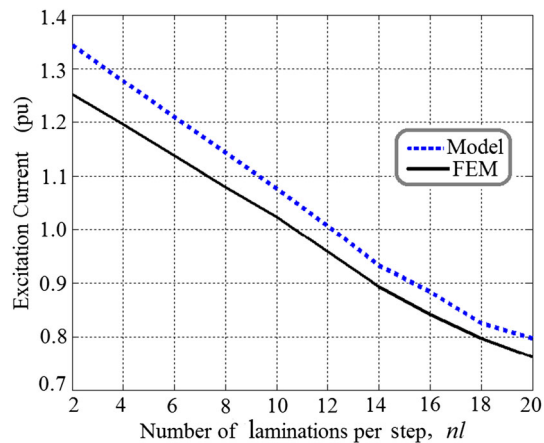


Figure 9. Excitation current results by applying FEM as well as the proposed model Equation (16) when the number of laminations per step (nl) is changing.

illustrates the application and feasibility of using the proposed model. It has been demonstrated that the proposed model simplifies the problem without sacrificing the accuracy. The results obtained were compared with measurement values and the differences were small; the biggest difference was 8% for the case of excitation current. The model has been validated in a single-phase shell-type transformer as well as in a three-phase transformer. Besides the usefulness of the model to compute the component losses, it could be also used as a core design tool since it provides important information such as the effects of the joint zone or the impact of the number of laminations per step on core losses. Consequently, the present research work is very useful for the design and manufacturing of distribution transformers.

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APPENDIX

FEM SIMULATION DETAILS

The formulation for computing eddy current losses in each lamination using FEM is given by

$$P_e = \frac{1}{2} \cdot \text{Re} \left[\sum_{i=1}^n (\rho_i \cdot J_{e_i}^* \cdot J_{e_i}) \cdot V_i \right] \quad (\text{A1})$$

where n represents the number of finite elements for the laminated core, ρ_i is a diagonal tensor of resistivity of the GOSS, J_{e_i} is the eddy current density vector of the finite element i , $J_{e_i}^*$ is its conjugate, and V_i is the volume of the element i . The eddy current density is given by:

$$Je_i = -j\sigma \cdot \omega \cdot \mathbf{A}_i = -\sigma \cdot \frac{1}{n} \cdot \sum_{i=n}^n \mathbf{N}_A^T \cdot \mathbf{A}_i \quad (\text{A2})$$

where \mathbf{N}_A represents the element shape functions for the vector potential \mathbf{A} .

The executed FEM simulations solve the quasi-static magnetic formulation in the frequency domain (in our case only for 60 Hz). The simulations were done in 2D and 3D: for the 2D case, all the laminations were modeled, while for the 3D case, it was necessary to increase the lamination thickness with the aim to reduce the number of laminations to be simulated, that is why it was necessary to find an equivalent conductivity as in [12] for the new lamination thickness.

The core and the primary windings were enclosed by the tank represented by a cylindrical object in 3D and by a rectangular object in 2D. The tank walls represent the external boundaries, which define the magnetic insulation boundary. For the 2D simulations about 300,000 triangular finite elements were employed spending about 6 GB of RAM memory for the solution. For the 3D simulations, about 400,000 tetrahedral elements were employed using approximately 10 GB of RAM memory for the solution.